



Notes, Comments, and Letters to the Editor

# Communication in games of incomplete information: Two players

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## Abstract

We study the effect of communication in two-person games of incomplete information. We show that for any two-player game of incomplete information, any rational mediated communication mechanism outcome (satisfying a Nash domination condition) can be implemented as the perfect Bayesian equilibrium of a cheap-communication extension of the original game.

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## 1. Introduction

Since the seminal paper of Crawford and Sobel [5], it has been well understood that communication in games of incomplete information can expand the set of equilibrium outcomes that can be achieved and, in some instances, even provide the players with Pareto improving outcomes. In this paper, we examine the question of how much the set of equilibrium outcomes can be expanded via cheap unmediated communication procedures in two-person games of incomplete information.

In order to put our results into perspective, consider the notion of mediated communication mechanisms. These mechanisms have a disinterested third party, known as a *mediator*, to whom the players report their private information who then makes private recommendations to the two players. The Bayesian Nash equilibria of this game are known as mediated communication mechanism outcomes. By the Revelation Principle, we can restrict attention to strategies where both

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players report their types honestly and are obedient in following the mediator's recommendations. Following Myerson [12, pp. 261–262], we say that “. . .incentive-compatible mediation plans (mediation communication mechanisms) are the appropriate generalisation of the concept of correlated equilibria in Bayesian games with communication.” These outcomes are also referred to as *communication equilibrium outcomes* or CEOs. The CEOs form a benchmark because any outcome that is not a CEO is not achievable via any incentive compatible communication mechanism.

But just how important is the presence of the mediator if the players are interested in expanding the set of Bayesian Nash equilibria of an incomplete information game through a cheap-communication procedure? In other words, What CEOs can be realised as a perfect Bayesian equilibrium of a cheap-communication extension of the original game? (Recall that a cheap-communication extension of a Bayesian game  $G$  is an extensive form game where players are allowed to communicate costlessly before playing the original game  $G$ .) For the case of five or more players, Gerardi [9] shows that in any finite Bayesian game with full support, any rational CEO can be implemented as a sequential equilibrium of a cheap-talk extension of the original game. This result was improved upon by Ben-Porath [4] who shows that in any finite Bayesian game with three or more players and full support, any rational CEO (satisfying a Nash domination condition) can be implemented as a sequential equilibrium of a cheap-talk extension of the original game. Note that both Gerardi and Ben-Porath use communication protocols that only rely on verbal communication.

In this paper, we consider two-player finite games of incomplete information. We characterise the outcomes implementable using communication mechanisms that are cheap and unmediated. In our main theorem below, we show that any (rational) CEO of a two-person game that gives to every type of every player a payoff which is strictly greater than some payoff in the convex hull of the set of Bayesian Nash equilibrium payoffs of the original game can be implemented by a cheap-communication procedure. Our mechanism is an extension of Ben-Porath's [3] mechanism for two-person games of complete information which implements any rational correlated equilibrium in which Pareto dominates some Nash equilibrium. Before we sketch our mechanism, let us clarify what it is that we are not claiming. The mechanism we use is cheap and non-binding but it requires more than face-to-face talk. It requires that players be able to write messages in envelopes, open envelopes in private and verify the contents of the envelopes. We shall see in the sequel that these requirements are necessary for our result to hold.

Let us start with a description of our mechanism for complete information games (which, although based on Ben-Porath's [3] mechanism, is slightly different). Suppose the correlated equilibrium to be implemented consists of three action profiles  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ , each with probability  $\frac{1}{3}$  and suppose this correlated equilibrium Pareto dominates some Nash equilibrium. Our protocol works as follows. Player 2 prepares three unmarked envelopes, one for each action profile. In the envelope for the action profile  $(x_1, y_1)$ , Player 2 places two other envelopes marked *Player 1* and *Player 2*. The envelope marked *Player 1* contains the action  $x_1$  and the envelope marked *Player 2* contains the action  $y_1$ . The envelopes for the other action profiles are constructed similarly.

The players now conduct a joint lottery (see §2 for details) which requires them to verify the contents of the envelopes with some probability and not with complementary probability. Suppose the joint lottery does not require the players to verify the contents of the envelopes. Then Player 1 picks one of the unmarked envelopes at random and opens it. He takes the envelope marked *Player 1* and keeps it for himself and hands the other marked envelope to Player 2. Both players now open their envelopes in private and play the recommended action. Since the

recommendations follow a correlated equilibrium, it is optimal for both players to follow the recommendations.

If instead, the joint lottery requires them to verify the contents of the envelopes, Player 1 publicly verifies the contents. If there is any deviation from the prescription above, they play the Nash equilibrium which is Pareto dominated by the correlated equilibrium. If there is no deviation, then Player 2 prepares another set of envelopes according to the prescription above. To ensure that Player 2 does not have an incentive to deviate in the envelope making stage, we need to ensure that the probability of monitoring the contents of the envelopes is sufficiently high (but less than 1). Thus, the communication ends in finite time a.s.

Let us consider the case where Player 1 comes in many types. Then, Player 2 makes the corresponding envelopes for each type of Player 1 and gives Player 1 large envelopes marked  $t_1^i$  for each type  $i$  of Player 1. The contents of each envelope are exactly as described above. Player 1 takes the large envelope corresponding to his type (in private) and destroys all the others. He then gives Player 2 a recommendation and keeps one for himself. Once again, for the reasons outlined above, this is incentive compatible.

If Player 2 also comes in many types, he prepares envelopes for each of his possible types too and labels them by Player 1's type and some random permutation of his type. This ensures that in the random monitoring stage, his true type cannot be inferred. If the outcome of the monitoring lottery is that they do not verify the contents of the envelopes, he retains the envelopes corresponding to his true type (which remains unknown due to the permuting of the labels) and discards all the others. They then proceed as above, when there is only one type of Player 2.

At this stage, it is useful to point out why we need the envelopes. Recall that Forges [6] demonstrates a two-person example where unbounded cheap-talk (i.e. verbal, face-to-face communication) does not approximate the set of communication equilibria. (Also note that Forges uses the solution concept of Bayesian Nash equilibrium for this example.) From Theorem A of Aumann and Hart [1], we know that if this is true for the specific form of communication considered by Forges [6], then the result also holds for any other protocol that the players might consider as long as they restrict themselves to verbal communication. Thus, the envelopes, random monitoring and the players being able to make a selection from the envelopes in private are necessary<sup>1</sup> if we are to implement the set of communication equilibria.

To see what role the envelopes play, consider what one cannot do in verbal communication, namely, commit to a strategy. The envelopes allow the players to commit to taking some actions (conditional on the joint lottery not requiring verification) without disclosing any private information. Clearly, this is not possible in verbal communication.

Before we delve into the rest of the paper, we shall review the relevant literature. As mentioned before, cheap-talk in a class of Bayesian games was first shown to be useful by Crawford and Sobel [5] while mediated solutions are described in great detail in Myerson (Chapter 6 of [12]). A version of the mediated solution for extensive form games is described by Myerson in [11]. Bárány [2] studies how a communication protocol can implement rational correlated equilibria in games of complete information. He shows that if there are at least four players, then any rational correlated equilibrium can be implemented via a communication protocol. Forges [7] extends this study to Bayesian games. She shows that when there are at least four players, every ex ante mediated outcome can be implemented as a correlated equilibrium of a cheap talk extension of the original game. Now using the result of Bárány, it is straightforward to show that every rational

<sup>1</sup> Of course, the envelopes are merely a metaphor for any other devices, such as urns and balls, which can play a similar role.

ex ante mediated outcome can be implemented as a Nash equilibrium of a cheap-communication extension of the original game. Here, ex ante means that all the communication occurs before the players learn their types. Gerardi [8] extends this result to the standard interim case, where players communicate after learning their types. Unfortunately, none of these results use procedures that are sequentially rational.

The question of sequential rationality was addressed by Gerardi [9] for five or more players and Ben-Porath [4] for three or more players (whose results have been described above). In the context of mediated solutions, we should also mention a result due to Lehrer and Sorin [10]. They point out that in general, mediated solutions need not be stochastic. They introduce a deterministic mediated mechanism which can implement any rational mediated outcome (which, in general, is not deterministic). Their result applies to both complete information games (wherein they implement correlated equilibria) and Bayesian games. It should be noted that the mechanism we use in the proof of our main theorem in §3 is as much a descendant of the Lehrer–Sorin mechanism as of the Ben-Porath [3] mechanism.

Finally, Aumann and Hart [1] completely characterise the set of equilibrium outcomes from unbounded verbal communication in two-player games of one-sided incomplete information. Needless to say, this set is usually smaller than the set of CEOs. The remainder of the paper is structured as follows. In §2 we introduce the model, all the relevant definitions and our main theorem. In §3 we prove our main theorem.

## 2. The model and main result

A Bayesian game is characterised by  $\Gamma^b := (N, (C_i)_{i \in N}, (T_i)_{i \in N}, P, (u_i)_{i \in N})$ , where  $N$  is a (finite) set of players and for each player  $i \in N$ , the set of possible actions is  $C_i$ , the set of possible types is  $T_i$  and  $i$ 's utility is given by  $u_i$ . If we let  $C := \times_i C_i$  and  $T := \times_i T_i$ , then  $u_i : C \times T \rightarrow \mathbb{R}$  is a von Neumann–Morgenstern utility function and  $P$  is a probability measure on  $T$ . We say that  $\Gamma^b$  is finite if  $C_i$  and  $T_i$  are finite for each  $i \in N$ . (We will mainly be interested in the case where  $N = 2$ .)

Each player  $i$  first learns his own type and then forms beliefs over the types of the other players. His belief over the types of the other players is denoted by  $p_i : T_i \rightarrow \Delta(T_{-i})$  where  $\Delta(T_i)$  is the set of probability distributions over  $T_{-i}$  and is calculated via Bayes' rule so that

$$p_i(t_{-i}|t_i) := \frac{P(t)}{\sum_{s_{-i} \in T_{-i}} P(s_{-i}, t_i)}$$

Thus, we can also say that beliefs are consistent since all the conditional beliefs  $p_i$  can be derived from a single probability measure  $P$ . We shall say that the game has full support if, for each  $t \in T$ ,  $P(t) > 0$ .

Player  $i$ 's strategy  $\sigma_i$  is a function  $\sigma_i : T_i \rightarrow \Delta(C_i)$ . A profile of strategies  $\sigma := (\sigma_i)_{i \in N}$  is a Bayesian Nash equilibrium if for each player  $i$ , given his type  $t_i \in T_i$ ,  $\sigma_i(t_i)$  maximises his expected utility given his information and the other players' strategies. In other words,

$$\sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) u_i(\sigma_i(t_i), \sigma_{-i}(t_{-i}), t) \geq \sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) u_i(\tilde{\sigma}_i(t_i), \sigma_{-i}(t_{-i}), t)$$

for all strategies  $\tilde{\sigma}_i \in \Delta(C_i)^{T_i}$ .<sup>2</sup>

<sup>2</sup> If  $X$  and  $Y$  are sets,  $Y^X$  denotes the set of all mappings from  $X$  to  $Y$ .

Now suppose there is a disinterested mediator with whom the players can communicate. This gives us a Bayesian game with communication wherein the players communicate after they learn their types but before they choose an action. Let each player report his type confidentially to the mediator. The mediator, upon receiving messages from all the players, makes private probabilistic recommendations of actions to take to each player. Specifically, player  $i$  sends a message  $m_i \in T_i$  to the mediator. Then the mediator makes a recommendation according to  $\mu : T \rightarrow \Delta(C)$ . Thus, a strategy for player  $i$  is a pair  $(m_i, \delta_i)$  where  $m_i$  is the type report that player  $i$  sends to the mediator and  $\delta_i : C_i \rightarrow \Delta(C_i)$  is such that player  $i$ , upon receiving the recommendation  $c_i$ , takes the action  $\delta_i(c_i)$ . We will require that  $\sum_{\tilde{c} \in C} \mu(\tilde{c}|t) = 1$  and  $\mu(c|t) \geq 0$  for all  $c \in C$  and  $t \in T$ . Indeed, any such function  $\mu : T \rightarrow \Delta(C)$  will be referred to as a *mediated communication mechanism*. Note that the mediated communication mechanism is cheap, non-binding and non-verifiable. We shall call a mediated communication mechanism *rational* if  $\mu(c|t)$  is rational for each  $c \in C$  and for every  $t \in T$ .

Let us consider the instance where each player reports his type honestly to the mediator and obeys the recommendation of the mediator. Then, player  $i$ 's expected utility from the mediation mechanism  $\mu$  is

$$U_i(\mu|t_i) := \sum_{t_{-i} \in T_{-i}} \sum_{c \in C} p_i(t_{-i}|t_i) \mu(c|t) u_i(c, t).$$

It goes without saying that we can restrict attention to truth-telling strategies in the reporting stage precisely because of the Revelation Principle (see, for instance, [12]). The mediation plan  $\mu$  therefore induces a communication game  $\Gamma_\mu^b$  wherein each player chooses a reporting strategy  $m_i(t_i)$  and a choice of action  $\delta_i(c_i)$  upon receipt of the recommendation  $c_i$ . Thus, a strategy is a pair  $(m_i, \delta_i)$ . The type sets are the same as in  $\Gamma^b$  and the utility functions in  $\Gamma_\mu^b$  are derived in the obvious way. If player  $i$  were to use strategy  $(m_i, \delta_i)$ , his expected utility from the mechanism  $\mu$  is given by

$$U_i(\mu, \delta_i, m_i|t_i) := \sum_{t_{-i} \in T_{-i}} \sum_{c \in C} p_i(t_{-i}|t_i) \mu(c|t_{-i}, m_i) u_i(c_{-i}, \delta_i(c_i), t),$$

if all the other players remain honest and obedient. We will say that a mediated communication mechanism  $\mu$  is *incentive compatible* if and only if being honest while reporting their type to the mediator and obedient while following the recommendation of the mediator is a Bayesian Nash equilibrium of the game with communication. Thus,  $\mu$  is incentive compatible if, for all  $i \in N$ ,  $t_i \in T_i$ ,  $m_i \in T_i$  (the report of player  $i$ ) and  $\delta_i \in \Delta(C_i)^{C_i}$  (the choice of action for player  $i$  when the recommendation is  $c_i$ ), it is the case that

$$U_i(\mu|t) \geq U_i(\mu, \delta_i, m_i|t_i).$$

The mediator represents, in an indirect way, commitment among the players. Thus, the set of incentive compatible mechanisms represents all the outcomes that could potentially be realised as the equilibrium of any other mechanism or protocol.

Recall that an outcome function is a function  $\psi : T \rightarrow \Delta(C)$ . (Then  $\psi(t)$  is a probability measure on  $C$  and  $\psi(t)(c)$  is the probability of the outcome being  $c$ .) We will call  $\psi$  a *communication equilibrium outcome* (CEO) if it is the outcome of some mediated communication mechanism. Consider now any other cheap (unmediated) communication extension of  $\Gamma^b$  represented by

$\Gamma_c^b$ . Here,  $\Gamma_c^b$  is an extensive form game where players communicate after learning their types. At some endogenously determined point in time, the players simultaneously choose their actions from the original game  $\Gamma^b$ . A strategy profile  $\sigma = (\sigma_i)_{i \in N}$  in  $\Gamma_c^b$  induces an outcome function  $\psi^\sigma : T \rightarrow \Delta(C)$ .

As mentioned above, the set of mediated communication mechanism outcomes encompasses everything that is achievable as the equilibrium outcome of some mechanism. We are interested in implementing these mediated outcomes via communication mechanisms that are unmediated. In other words, our goal is to find a mechanism so that for any mediated communication mechanism outcome, there is a perfect Bayesian equilibrium of the game induced by the mechanism that implements the mediated mechanism. We need one more definition before we continue. For any Bayesian game  $\Gamma^b$ , let  $NE(\Gamma^b)$  be the set of Bayesian Nash equilibrium payoffs to each type of each player and  $\text{conv}(NE(\Gamma^b))$  its convex hull.

**Definition 2.1.** Let  $\Gamma^b$  be a Bayesian game. A mediated communication mechanism  $\mu$  is *Nash dominating* if there exists a payoff vector  $\alpha \in \text{conv}(NE(\Gamma^b))$  with payoff  $\alpha_m^j$  to type  $t_j$  of player  $m$  such that  $U_m(\mu|t_j) > \alpha_m^j$  for all  $t_j \in T_m$  and all  $m \in N$ . We shall call such an equilibrium a Nash dominated equilibrium.

We are now ready to state our main theorem.

**Main theorem.** Let  $\Gamma^b$  be any Bayesian game with two players and full support and let  $\psi$  be an outcome function that is induced by some rational, Nash dominating, mediated communication mechanism  $\mu$ . Then, there exists an unmediated communication mechanism that is cheap and non-binding that implements  $\psi$  with probability 1.

We will need a few more ideas in order to move towards our end. We will use the idea of a *joint lottery*. A joint lottery simulates a public randomising device and works in a straightforward manner for lotteries of the form  $(\lambda, 1 - \lambda)$  where  $\lambda \in \mathbb{Q} \cap [0, 1]$ . Consider the case where there are two players and suppose they wish to pursue a particular course of action called outcome 1 with (rational) probability  $\lambda$  and some other outcome, 2, with the complementary probability. To achieve this, we assume that  $\lambda = \frac{p}{q}$  where  $p, q \in \mathbb{Z}_+$  and are coprime. Now let each player have  $\mathbb{Z}_q := \mathbb{Z}/q\mathbb{Z} = \{0, \dots, q - 1\}$  as a message space. The players simultaneously send messages  $z_1$  and  $z_2$ , respectively, to each other from the message space with each message being drawn according to the uniform distribution. Then, we will stipulate that outcome 1 obtains if  $z_1 + z_2 \pmod{q} \in \mathbb{Z}_p$ . Note that  $z_1 + z_2 \pmod{q}$  is uniformly distributed over  $\mathbb{Z}_q$ . It is now easy to see that outcome 1 obtains with probability  $\lambda = \frac{p}{q}$  and it is also the case that neither player has an incentive to unilaterally choose the message according to some other distribution (which constitutes a change in strategy).

We will also use the coding tools of envelopes with notes in them and larger envelopes which contain these envelopes. (These correspond, respectively, to the balls with messages in them and the urns containing the balls in Ben-Porath [3]. Our choice of this alternative terminology stems from aesthetic concerns as we will be using envelopes within envelopes (which we shall refer to as *nested envelopes*) and we find the image of urns within urns somewhat unappealing.) The nested envelopes provide the players with the possibility for one player to make a hidden selection. This enables them to transmit as much information as is achieved with a mediator and no more. (To see how, see §3 below.) We are also implicitly assuming here that Player 2 (say) does not have access to a technology which allows him to mark the envelopes in a manner which is undetectable

by Player 1. This is important because Player 1 needs to be sure that he does not inadvertently reveal his type when he hands Player 2 an envelope.<sup>3</sup>

Let us now consider how the players might achieve a Bayesian Nash equilibrium which gives payoffs  $\alpha \in \text{conv}(\text{NE}(I^b))$ , without the use of envelopes. In other words, we shall construct a perfect bayesian equilibrium of a proper subgame which gives expected value  $\alpha_m^j$  to type  $j$  of player  $m$  through verbal communication alone. For simplicity, assume  $\alpha$  is the convex combination of only two points in  $\text{NE}(I^b)$ . (By repeating this procedure finitely many times, it is possible to achieve arbitrary  $\alpha \in \text{conv}(\text{NE}(I^b))$ ). In particular, suppose that  $\alpha$  can be achieved by playing one Bayesian Nash equilibrium with probability  $\rho \in [0, 1]$  and another with probability  $1 - \rho$ . This can be achieved by conducting a lottery with the respective probabilities. But the procedure described above, namely the joint lotteries, only works for rational probabilities (since our message space is finite). Nevertheless, a multi-step version of the procedure can be adopted to produce any probability.

Let each player have  $\{a, b\}$  as a message set. With probability  $\frac{1}{2}$ , the players simultaneously send the message  $a$  to each other. Then, let the players record the outcome as 0 if they send each other the same message and 1 if the messages are different. Repeat this procedure again. After  $n$  repetitions, the players will have generated a string of 0's and 1's, which can be written as  $0.o_1o_2 \dots o_n$  (where  $o_i \in \{0, 1\}$ ). We will regard this as the binary expansion of a real number. The players will stop and implement the first outcome if the number they generate is less than  $\rho$  and the second outcome if the number is greater than  $\rho$ . Also, they will stop in finite time a.s. For example, consider the case where  $\rho = 0.0101010101 \dots$  in base 2. Then, there will be a first digit which does not match the expansion. In particular, suppose after four rounds of communication, the players generate 0.0100, they will stop and play the first outcome. They could not have stopped after three rounds as the first three digits of the number they generated were the same as the first three digits in the binary expansion of  $\rho$ . But the fourth round told them that the number they were generating was definitely less than  $\rho$ .

This gives us a procedure to implement arbitrary payoffs in  $\text{conv}(\text{NE}(I^b))$ . Note that the participation of *both* players is necessary in order to produce the required convex combination. Also, note that the procedure is perfect bayesian as at no point in time does either player have an incentive to deviate to another strategy. (Changing the probabilities of choosing  $a$  in any stage does not alter the probability of the outcomes  $aa$  or  $bb$ .)

### 3. Proof of main theorem

Let us assume that the players wish to implement a mediated solution  $\mu$ , as described in §2.

*The mechanism:*

*Step 1.* Player 2 makes envelopes for all states  $t \in T$ . An envelope for state  $t = (t_i, \tau_j)$  contains several *unmarked* envelopes. Each unmarked envelope contains a recommendation for an action profile. The recommendations are made via two smaller envelopes marked *Player 1* and *Player 2*. The envelope marked *Player 1* contains a recommendation for Player 1 to play and similarly for the envelope marked *Player 2*. The fraction of unmarked envelopes which recommend the action profile  $c$  is  $\mu(c|t)$ . Now the players proceed to Step 2.

*Step 2.* This is the random monitoring stage. The players conduct a joint lottery where with probability  $\rho$ , the contents of all the envelopes are checked. If there is a deviation by Player 2, i.e. if he has not placed the messages in the envelopes and the nested envelopes according to the

<sup>3</sup> I would like to thank an anonymous referee for pointing out this implicit assumption.

mediated solution  $\mu$ , they play the Nash dominated equilibrium. If there is no deviation, they go back to Step 1. If the joint lottery does not require them to open all the envelopes, the players proceed to Step 3.

*Step 3.* Suppose that Player 2's true type is  $\tau_1$ . He now removes all the envelopes corresponding to states  $t = (t_i, \tau_j)$  where  $\tau_j \neq \tau_1$  and hands the remaining envelopes to Player 1. (In other words, he tells Player 1 that he is of type  $m = \xi(\tau_1)$ , but since Player 1 does not know  $\xi$  and all the  $\xi$ 's are equally likely, his beliefs about Player 2's type remain unchanged.) Player 1 now picks the envelope corresponding to his type and picks one of the envelopes in it. This envelope contains the enveloped marked *Player 1* and *Player 2*. He hands Player 2 his envelope and they play the recommended action.

The reason for Player 2 making envelopes corresponding to all states  $t \in T$  is that if he just made it for all  $t = (t_i, \tau_j)$  where  $\tau_j$  is his true type and the players are required to inspect the contents of the envelopes in the monitoring stage, then Player 1 can, in principle, infer the type of Player 2.

All that remains to be done is to determine a monitoring probability,  $\rho$ . Recall that  $\alpha_m^j$  is the payoff to type  $\tau_j$  of player  $m$ . Now consider the case where type  $\tau_j$  of player  $m$  deviates. Then, assuming that the maximum payoff in the game is  $W$ , his payoffs are bounded from above by

$$\rho\alpha_m^j + (1 - \rho)W.$$

But his expected payoff from following the protocol is  $U_m(\mu|\tau_j)$  and we require that

$$U_m(\mu|\tau_j) > \rho\alpha_m^j + (1 - \rho)W. \quad (1)$$

But  $U_m(\mu|\tau_j) > \alpha_m^j$  by assumption, therefore there exists a  $\bar{\rho}$  such that (1) is satisfied and we can find a  $\rho \geq \bar{\rho}$  such that  $\rho \in \mathbb{Q} \cap [0, 1]$ . We now consider the issue of perfection. The information sets of the equilibrium path are ones where Player 2 deviates in the making of the envelopes. Since the deviations do not provide any information about Player 2's type, the priors remain the same and the players then proceed to play the Nash dominated equilibrium. Along the equilibrium path, the players' priors change when they receive a recommendation. But since we are on the equilibrium path, the posterior beliefs are the same as those that would have resulted from the recommendations being made by a mediator. If there is a deviation by Player 2, the players play the Nash dominated equilibrium and as seen above, this is perfect Bayesian in the subgame. Thus, the equilibrium constructed is perfect Bayesian.

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