Optimal Monitoring in Dynamic Financial Contracts

Arash Fahim [1] **Simon Gervais [2]** **R. Vijay Krishna [3]**

[1] Florida State University – Math [2] Duke University – Fuqua [3] Florida State University – Econ

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Firm Financing

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The *Three Ms* of Malfeasance

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- Applies generally:
  - ***VC financing today***
  - ***Publicly Traded Corporations***

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Milgrom-Roberts

output $\quad y = a + \sigma\varepsilon$

wage $\quad w = s_0 + \beta y$

Optimal (a, σ, β) jointly determined

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  - Comparative statics of risk aversion

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- Empirical literature on Governance ...

Model

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[follow DeMarzo and Sannikov (2006)]

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- Principal covers operating losses

[follow DeMarzo and Sannikov (2006)]

Model (contd)

- Firm produces cash flows

$$dY_t = \mu dt + \sigma_t dB_t$$

In DS: Σ is singleton

  - Volatility σ_t chosen by Principal at Cost $\rho(\sigma_t)$
  - $\sigma_t \in \Sigma = \{\sigma_{(0)}, \ldots, \sigma_{(n)}\}, \quad \sigma_{(i)} > \sigma_{(i+1)}$
  - $\rho(\sigma_{(i)}) < \rho(\sigma_{(i+1)})$: More accuracy is costlier

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- Principal observes Agent ***report*** $\hat{Y}_t$ where

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- $D_t \geqslant 0$ is ***cash-flow diversion***

In DS: Σ is singleton

- Benefit of diversion D_t is λD_t, where $\lambda \in (0, 1]$
- Always optimal to implement truth-telling: $D_t = 0$ for all $t \geqslant 0$

Model (contd)

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Recap

Cash Flow

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Agent Reports $\hat{Y}_t$

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Alternative Model

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  - $\sigma = (\sigma_t)$: Monitoring levels

$$\text{Profit} = F(w = w_0; \Phi) := \mathbf{E}^{D=0,\sigma}\left[\int_0^\tau e^{-rt}\left[(\mu - \rho(\sigma_t))dt - dC_t\right]\right]$$

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$$\mathbf{E}^{D=0,\sigma}\left[\int_0^\tau e^{-\gamma t} dC_t\right] \geqslant \mathbf{E}^{D,\sigma}\left[\int_0^\tau e^{-\gamma t}(dC_t + \lambda D_t dt)\right]$$

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  - works because output is BM, iid increments

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- ***IC*** $\iff$ ***Gain*** $\leqslant$ ***Loss***

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subject to (i) IC and (ii) PK

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F is concave and C^2 solution of (variational) HJB …

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- F is C^2 solution [Schauder theory]

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- ***Backloaded payments:*** C_t satisfies

$$C_t=\int_0^t 1(W_s\geqslant w^\star)dC_s$$

Optimal Contract

$$dW_t = \gamma W_t dt - dC_t + \lambda \sigma_t dB_t$$

$w^\star < \infty$ if and only if $\gamma > r$

- $w^\star$ is payment boundary
- $W_t \in [0, w^\star]$
- $\tau = \inf\{t : W_t = 0\} < \infty$
- $dC = 0$ for $w \in [0, w^\star)$ (***backload***)
- C_t satisfies

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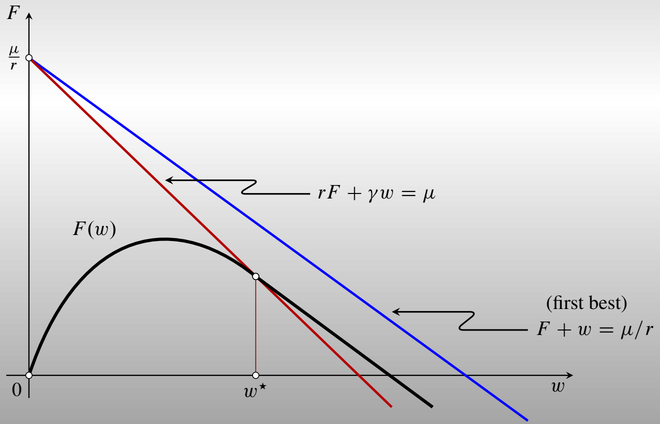

Optimal Contract: Monitoring

$$rF(w) = \mu + \gamma w F'(w) + \underbrace{\max_{C}\left[-\left(F'(w)+1\right)dC\right]}_{=0} + \max_{z \geqslant \lambda\sigma, \sigma \in \Sigma}\left[\tfrac{1}{2}z^2 F''(w) - \rho(\sigma)\right]$$

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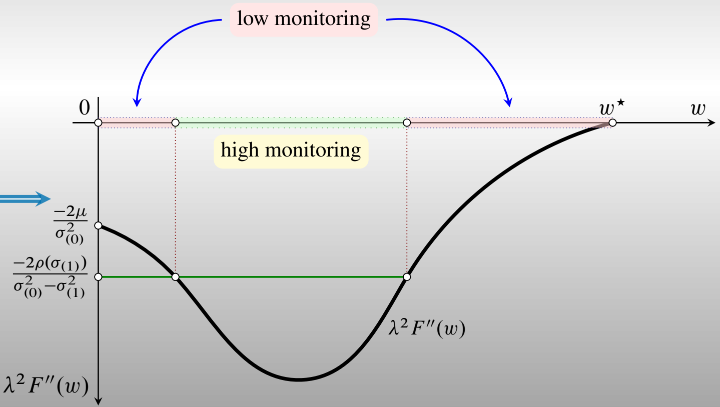

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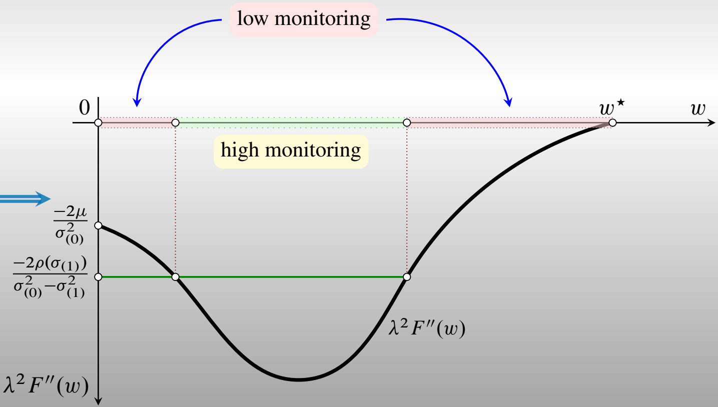

Pay Sensitivity and Monitoring are ***substitutes***

Shape of Risk Aversion

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$$\text{Risk Aversion} = -\lambda^2 F''(w) \quad \propto \quad \underbrace{r\left[\frac{\mu}{r} - F(w) - w\right]}_{\text{efficiency loss} \geqslant 0} + \underbrace{\gamma w\left(F'(w) + r/\gamma\right)}_{\text{expected change in value}}$$

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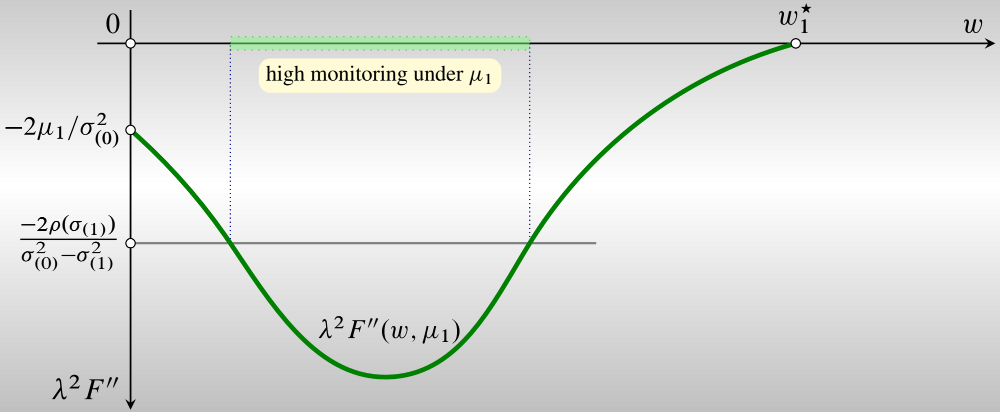
Proposition: There exists $\mu^\dagger$ such that

$$\mu \geqslant \mu^\dagger \implies \begin{cases} F'(0;\mu) \geqslant 0 \\ \text{RA} \uparrow \text{ then } \downarrow \end{cases}$$

$$\mu < \mu^\dagger \implies \begin{cases} F'(0;\mu) < 0 \\ \text{RA decreases} \end{cases}$$

Comparative Statics: Risk Aversion when $\mu_1 < \mu_2$



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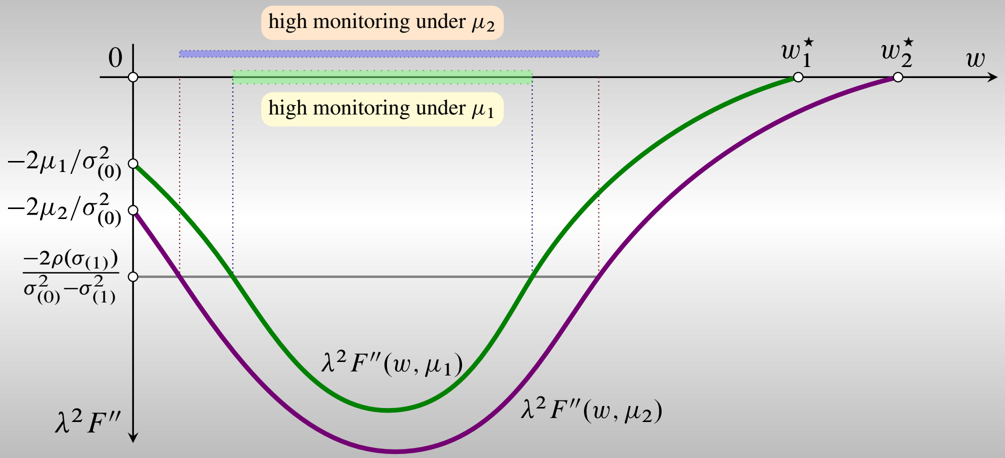

Comparative Statics: Intuition

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For fixed $w > 0$,

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Efficiency loss increasing in μ

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$\Longrightarrow$ (i) Risk Aversion increases in μ (ii) w and μ are ***complements***

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Follow BMPR

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Securities and Assets

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- Stock pays ***dividend*** $\lambda^{-1}dC$ when $M_t = w^\star/\lambda$ [agent controls dividends]

Stock Price

$$S_t = \mathbf{E}_t\left[\int_t^\tau e^{-r(s-t)}\lambda^{-1}dC_s\right]$$

but $S_t = \mathcal{S}(M_t)$, so

$$dS_t = rS_t dt + \mathrm{V}_t dB_t - \lambda^{-1}dC_t$$

where $\mathrm{V}_t = \mathcal{S}'(M_t)\sigma(\lambda M_t)/S_t$ = local volatility

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BVP for stock prices

$$r\mathcal{S}(m) = \gamma m\mathcal{S}'(m) + \tfrac{1}{2}\sigma^2(\lambda m)\mathcal{S}''(m)$$

- $\mathcal{S}(0) = 0$
- $\mathcal{S}'(w^*/\lambda) = 1$

Then, $S_t = \mathcal{S}(M_t)$ and $\mathcal{S}(\cdot)$ is

- strictly increasing
- strictly concave $\implies$ continuous
- C^2 except at finitely many points
  even though $\sigma(\cdot)$ discontinuous!

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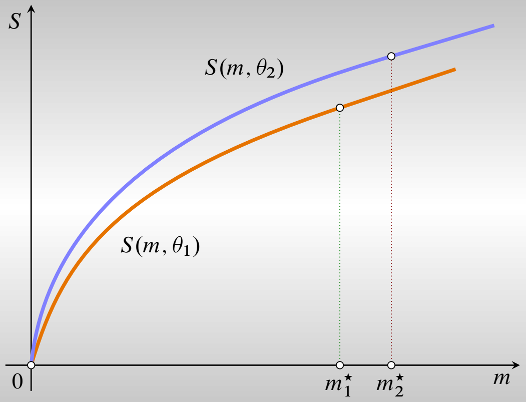

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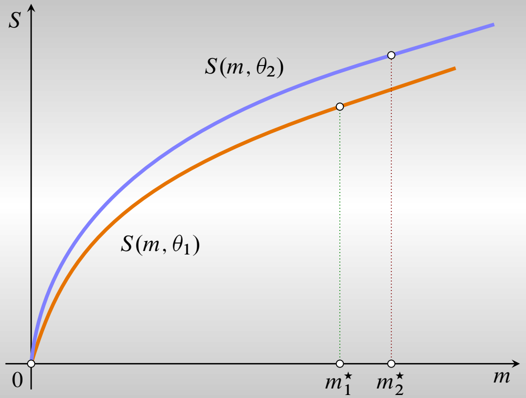

Dynamics: Monitoring increases after drop in stock price (Vafeas 1999)

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Proposition

$$\underbrace{(1-\lambda)S_t + D_t}_{\text{market value}} > \underbrace{F(\lambda M_t) + M_t}_{\text{true value}}$$

Difference = $\mathbf{E}_t \left[\int_t^{\tau} e^{-r(s-t)} \rho(\sigma_s) ds \right]$ = monitoring costs

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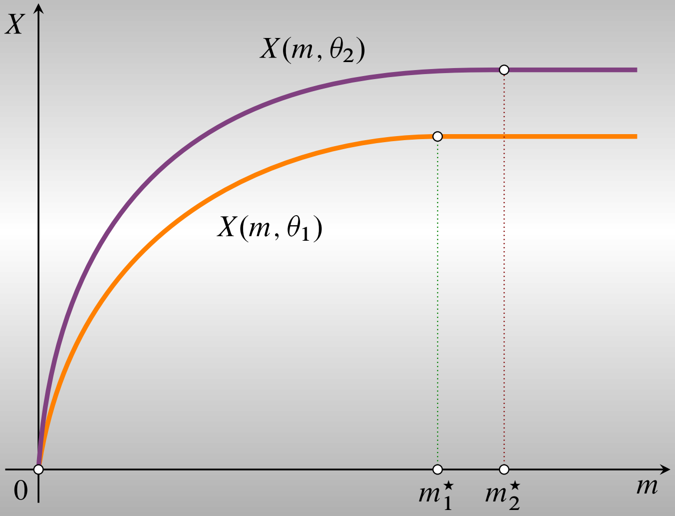

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Monitoring Intensity — General Moral Hazard

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Doesn't require F, G to be C^2;
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Thank you!

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Effect of SOX

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- Δ_t measured in many ways

$$\underbrace{V_t \cdot \Delta_t \cdot S_t}_{\text{local governance}} = \lambda \sigma_t$$

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